Innovation and Taxation at Start-up Firms

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I. Introduction

Cutting income taxes on profits from entrepreneurial or innovative behavior will certainly produce an increase in entrepreneurship and innovation at private business firms-–right?[1] It seems at first straightforward: Providing income tax incentives will increase the after-tax profits of the entrepreneur and therefore increase the motivation to engage in entrepreneurial or innovative activity.[2] But this standard analysis does not apply for start-ups and other capital-constrained firms that face a low probability of success.[3]

Instead, as we show in this Article, capital-constrained start-ups will only use conventional income tax breaks in the unlikely event that they succeed and become profitable. Meanwhile, sums invested in planning to reduce income taxes are not available for core business spending. Therefore, a start-up considering income tax planning must balance, on one hand, the advantage of a reduction in tax due on any

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[1] See generally E. Cary Brown, Tax Incentives for Investment, 52 Am. Econ. Rev. 335, 338 (1962) (explaining that "one can cut the rate of income tax to equalize the effect of [an] interest rate cut" and similarly affect the calculation of the net present value of an investment).

[2] See, e.g., Jacob Nussim & Avraham Tabbach, Tax-Loss Mechanisms, 81 U. Chi. L. Rev. 1509, 1544-45 (2014) (arguing generally that offset, transferability, or refundability rules for tax losses can achieve similar results and admitting only a small exception for a "last-period loss" under an offset regime); Nirupama Rao, Do Tax Credits Stimulate R&D Spending? The Effect of the R&D Tax Credit in Its First Decade 10 (N.Y.U. Wagner Sch. Pub. Serv., Working Paper No. 2272174, 2015), http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2272174 (developing a user cost of capital for the purchase of research that allows for delay in the use of research and development ("R&D") credits due in part to accumulated losses, but assumes that the credits eventually will be used).

[3] The lack of fit between the start-up case and the general analysis has long been recognized. See, e.g., Brown, note 1, at 336-37 ("The small firm, for which the particular investment may bulk large, could find itself faced with some considerable uncertainty about the value of prospective depreciation deductions.").

future profit against, on the other hand, the disadvantage of reducing business spending, which, as a result of the capital constraint, reduces the firm's expected probability of success.

In this Article we develop a model that shows conditions under which start-ups have no incentive to invest in tax planning. We suggest that, under reasonable assumptions for endowment, burn rate, and probability of success over time, a new start-up would rationally decide not to make a material investment in tax planning to eliminate income tax on any future profits. Later in a firm's life, when it has a larger endowment and a higher probability of success, the analysis shifts and there is a greater incentive to invest in tax planning.

Our argument that income tax breaks for entrepreneurship or innovation should not be expected to motivate the desired behavior at new start-up firms contrasts with frequent and broad recommendations for the adoption of such tax breaks.[4] Daring and ambitious start-up businesses, of the sort that have fueled venture capital ("VC") profits and built Silicon Valley's brand,[5] are more likely to embrace new or risky projects with possibly enormous payoffs of global scope. This probably means that start-ups are more likely to embrace innovation.[6] Policymakers may wish to encourage such innovation by start-ups even more. But income tax incentives that require planning invest-

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[4] See, e.g., George F. Allen, Maintaining America's Lead as the World Capital of Innovation, 1 Va. L. & Bus. Rev. 3, 4 (2006) (advocating "mak[ing] research and development tax credits permanent"); W. Wesley Hill & J. Sims Rhyne, III, Opening Pandora's Patent Box: Global Intellectual Property Tax Incentives and Their Implications for the United States, 53 Intell. Prop. L. Rev. 371, 405-06 (2013) ("To counteract [intellectual property] migration and increase innovation-based competitiveness in the United States, the [Information Technology and Innovation Foundation] proposes that the United States adopt a patent box combined with a redesigned R&D tax credit."); Bernard Knight & Goud Maragani, It Is Time for the United States to Implement a Patent Box Tax Regime to Encourage Domestic Manufacturing, 19 Stan. J. L. Bus. & Fin. 39, 41 (2013) (arguing that because "manufacturing itself provides the fuel for future innovation," the United States should "adopt a lower corporate income tax rate for" profits from "patented technology").

[5] Such ambitious start-ups do not make up the largest proportion of new or small businesses. See, e.g., Eric Hurst & Benjamin Wild Pugsley, What Do Small Businesses Do?, Brookings Papers on Econ. Activity, Fall 2011, at 73 (presenting data that show that most small businesses are small and often local service businesses with "little desire to grow big or to innovate in any observable way").

[6] A growing theoretical literature emphasizes the comparative advantage of VC financing for high-risk investments. See, e.g., Catherine Casamatta, Financing and Advising: Optimal Financial Contracts with Venture Capitalists, 58 J. Fin. 2059, 2072 (2003) (arguing that venture capitalists and entrepreneurs face a double moral hazard problem because the advice and effort of both is essential to the success of a risky start-up venture); Augustin Landier, Start-up Financing: From Banks to Venture Capital 4 (U. Chi. Graduate Sch. Bus., Working Paper, 2003), http://pages.stern.nyu.edu/~alandier/pdfs/bank_vc.pdf (arguing that where a high risk of failure is an accepted norm, investors are vulnerable to holdup by entrepreneurs and VC financing is designed to mitigate this holdup problem, for example through staged investment and control rights).

ment and that are directed at the early stages of a start-up's life are unlikely to successfully increase innovation and entrepreneurship.[7]

Our argument is also in tension with recent arguments that income tax breaks are sometimes superior to patent protection and other approaches to encouraging innovation. One attractive feature of income tax breaks is said to be their capacity to subsidize an innovative project earlier in time.[8] Another perceived advantage is that income tax breaks provide benefits even if the project fails.[9] It is said that these advantages are not available for leading alternative innovation-boosting policies, such as patent protection. But they are also not associated with income tax-reducing policies undertaken by new start-up firms. Start-up firms are generally not profitable for years after their founding.[10] Also, income tax assets often cannot be transferred. Thus, such benefits are often only valuable to a start-up if the firm itself becomes successful.[11]

Yet our model does not rely only on the uncertain and later-in-time value of income tax breaks. The motivating idea of the analysis we present is that for capital-constrained firms like start-ups, the probability of firm success is endogenous to, and inversely related to,

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[7] Some empirical evidence suggests that R&D tax credits, whose use is not as limited by early-stage decisions, increase R&D spending. See, e.g., Michael J. Graetz & Rachael Doud, Technological Innovation, International Competition, and the Challenges of International Income Taxation, 113 Colum. L. Rev. 347, 355-64 (2013) (summarizing research); Rao, note 2, at 15-24 (providing new evidence of effectiveness of credits based on tax return data using a sample of firms who claimed the credit at some point). See Stephen E. Shay, J. Clifton Fleming, Jr. & Robert J. Peroni, R&D Tax Incentives: Growth Panacea or Budget Trojan Horse?, 69 Tax L. Rev. 419 (2016) (questioning whether tax incentives for research and development are cost effective or targeted correctly).

[8] See Daniel J. Hemel & Lisa Larrimore Ouellette, Beyond the Patents-Prizes Debate, 92 Tex. L. Rev. 303, 311-13 (2013) (analyzing "stylized" "refundable tax credits" and explaining how they can have the same effect as direct government grants), 331-32 (describing patent boxes and acknowledging timing difference, but analyzing as equivalent to refundable tax credits). Hemel and Ouellette argue that other factors also support the use of tax credits in some circumstances, most prominently that, unlike government grants or prizes, but like patents, tax credits provide support when the innovator has an advantage over the government in identifying potentially successful projects. Id. at 375-78.

[9] See, e.g., Shawn P. Mahaffy, The Case for Tax: A Comparative Approach to Innovation Policy, 123 Yale L.J. 812, 818 (2013) (noting that tax "[s]ubsidizes experimentation" while patent protection "[s]ubsidizes success").

[10] See Eric J. Allen, The Information Content of the "Full" Deferred Tax Valuation Allowance for IPO Firms 1 (U.S.C. Levanthal Sch. Acct., Working Paper, 2012), http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2161340 (finding that 78% of VC-backed IPO firms have an NOL at the time of issuance).

[11] See, e.g., Alvin C. Warren, Jr. & Alan J. Auerbach, Transferability of Tax Incentives and the Fiction of Safe Harbor Leasing, 95 Harv. L. Rev. 1752, 1758-59 (1982) (explaining that depreciation deductions cannot provide the same benefit to an unprofitable start-up company as they do to a profitable company); Myron S. Scholes, Mark A. Wolfson, Merle Erickson, Michelle Hanlon, Edward L. Maydew & Terry Shevlin, Taxes and Business Strategy 126-27 (5th ed. 2015) (illustrating how the progressivity of the tax rate schedule disadvantages start-ups with net operating losses).

tax planning. This is because tax planning requires an investment. For example, taking advantage of a particular jurisdiction's patent box provisions may require that the firm incorporate in that jurisdiction. Such an incorporation choice is not free. It often costs more relative to other, cheaper incorporation alternatives.

The operating mechanism that we identify is that a tax planning investment reduces the endowment the firm can use for business planning, and consequently reduces the time the firm has to make it to the next stage of success. If a tax strategy's cost makes up a smaller portion of a firm's endowment, the firm loses less of its survival time if it invests in the tax strategy. In this case, its expected probability of success does not decrease as much, and it has a larger incentive to adopt the tax strategy. Because of a lasting capital constraint, a trade-off between tax planning and probability of success persists.

The first result of our model is that even if the additional cost of tax planning is modest, the investment may still be too rich for most start-ups' blood. Such firms' decision not to tax plan is a rational choice. The more a tax strategy costs relative to a firm's endowment, the smaller the firm's incentive to adopt the strategy. This means that many income tax incentives are unlikely to motivate the desired innovative or entrepreneurial behavior at early-stage start-up firms.

This first result squares with available empirical evidence that shows that a typical start-up firm does not tax plan early—between the moment of its founding and its "phase one" receipt of external financing such as VC financing.[12] A leading example of start-ups' failure to tax plan is their historic custom of organizing as C corporations, which cannot pass through tax losses to investors, rather than as LLCs, which can pass through tax losses.[13] Another example of start-ups' failure to tax plan is their pattern of organizing with parents that

[12] See Eric J. Allen & Sharat Raghavan, The Impact of Non-Tax Costs on the Tax Efficiency of Venture Capital Investment 21 (2011 Am. Tax. Assoc. Midyear Meeting Paper: JLTR Conference, Working Paper, 2011), http:// http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1759558 (finding that of 1067 firms that conducted IPOs between 1996 and 2011, only 55 had initially organized as LLCs and only 19 retained their LLC status at the time of IPO).

[13] See id. at 21; see also Allen, note 10, at 1 (finding that 78% of venture capital-backed IPOs have an NOL at the time of issuance); Joseph Bankman, The Structure of Silicon Valley Startups, 41 UCLA L. Rev. 1737, 1738 (1994) (noting puzzle of start-ups' C corporation organization despite their loss generation). Commentators are divided on the question of whether start-up investors could use losses. Compare Victor Fleischer, The Rational Exuberance of Structuring Venture Capital Start-Ups, 57 Tax L. Rev. 137, 153-55 (2003) (emphasizing that investors generally cannot use losses on a current basis), with Calvin H. Johnson, Why Do Venture Capital Funds Burn Research and Development Deductions, 29 Va. Tax. Rev. 29, 40 (2009) (noting that if C corporations invested in start-ups (directly or indirectly) they might benefit from start-up losses).

are U.S. C corporations, rather than as corporations located in a lower-tax jurisdiction.[14]

However, our model also shows that the tax planning investment is more attractive after a start-up attracts external investment, such as VC financing, and enjoys a larger endowment. Sometimes we call this second phase "phase two."[15] An income tax incentive that allows an investment in phase two rather than phase one, when the start-up has a larger endowment, has a greater chance of motivating the start-up to tax plan.

This second result, that a start-up has a greater incentive to tax plan in phase two, after it has received external financing, presents more of a tension with available empirical evidence. We do not know of data that shows that start-ups begin tax planning after they receive VC funding. A failure to tax plan may be because of strong market norms or path dependence, neither of which we consider in this Article. Yet even if our finding that a start-up has an incentive to tax plan in phase two conflicts with available data, it points out an opportunity for policymakers who advocate the use of income tax incentives to encourage start-up innovation and entrepreneurship. Our finding is also consistent with other work that suggests the promise of start-up tax planning strategies that preserve the option for later tax planning. The most prominent example of such a strategy is the initial organization of a start-up as an LLC.[16]

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14 Although structures that involve a U.S. parent and non-U.S. subsidiaries can also be tax-efficient, a non-U.S.-parent structure often offers even more tax planning opportunities, including expanded opportunities for U.S. base erosion. See, e.g., Mihir A. Desai & Dhammika Dharmapala, Do Strong Fences Make Strong Neighbors?, 63 Nat'l Tax J. 723, 724 (2010) (noting incentive for new firms to use a non-U.S. parent); Daniel Shaviro, The Rising Tax-Electivity of U.S. Corporate Residence, 64 Tax L. Rev. 377, 383-84 (2011) (noting that the degree of electivity turns on "nontax consequences"). Despite this, new U.S.-based companies with global ambitions overwhelmingly choose to incorporate in the United States. See Eric J. Allen & Susan C. Morse, Tax-Haven Incorporation for U.S.-Headquartered Firms: No Exodus Yet, 66 Nat'l Tax J. 395, 395 (2013) (reporting 27 instances of tax haven incorporation among 918 identified U.S.-headquartered multinational IPO firms). Of course, the organization location involves corporate governance and other nontax frictions. See Mitchell A. Kane & Edward B. Rock, Corporate Taxation and International Charter Competition, 106 Mich. L. Rev. 1229, 1239–40 (2008) (contrasting "corporate surplus" and "tax surplus" and challenging the idea that tax results should follow from corporate governance choice); Eric Talley, Corporate Inversions and the Unbundling of Regulatory Competition, 101 Va. L. Rev. 1649, 1653 (2015) (theorizing that the increased popularity of inversions may relate to the disassociation of corporate governance and tax results).

15 See notes 24-25 and accompanying text.

16 See Julie Berry Cullen & Roger H. Gordon, Taxes and Entrepreneurial Risk-Taking: Theory and Evidence for the U.S., 91 J. Pub. Econ 1479, 1480 (2007) (discussing the option value of a lower corporate tax rate on profits that can be claimed by later incorporation after initial organization as a pass-through in order to claim tax losses at higher individual rates).

Our focus on income tax benefits such as rate reductions encompasses the most important existing forms of income tax incentives specifically designed to encourage innovation.[17] Patent box provisions, which provide lower income tax rates for profit derived from specified innovative activity for corporations organized in the sponsoring jurisdiction, exactly fit the profile of the basic income tax incentive we analyze here.[18] Another frequently cited incentive is the R&D tax credit. Although such a credit theoretically could be refundable,[19] the existing R&D tax credit is not. Rather, it only can be used to reduce future income taxes payable.[20] Thus the R&D tax credit also fits the model we develop here.

In Part II we present the model that is the core of our argument. In Part III we develop assumptions about the values of the different variables in our model for start-up firms, based to the extent possible on available empirical data. In Part IV, we use the assumptions developed in Part III to illustrate our model and suggest the cost of tax planning that a start-up firm would accept in order to eliminate any income tax due on future profit.

II. Model

A. *Assumptions About Start-Up Firms*

Below, we sketch the assumptions on which our model is based.[21]

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[17] Other more systematic elements of the income tax system, such as rate schedules and the difference between corporate and personal income tax rates, may also impact the creation of new businesses in general, though effects are not restricted to firms that meet our definition of "start-up" firms. See, e.g., William M. Gentry & R. Glenn Hubbard, Tax Policy and Entrepreneurial Entry, 90 Am. Econ. Rev. 283, 283 (2000) (arguing and presenting empirical support for view that more sharply progressive personal income taxes discourage entrepreneurship because higher tax on income is not offset by higher tax benefit for losses); Tami Gurley-Calvez & Donald Bruce, Do Tax Cuts Promote Entrepreneurial Longevity, 61 Nat'l Tax J. 225, 240-41 (2008) (finding that decreased relative tax rates on "entrepreneurial activities" with certain tax filing statuses relative to employee compensation increased the duration of entrepreneurial effort).

[18] See, e.g., Graetz & Doud, note 7, at 363, 372-75 (defining patent boxes and discussing inconclusive evidence about the efficacy of patent box provisions' ability to increase technology investment and entrepreneurship).

[19] See Hemel & Ouellette, note 8, at 311-13 (analyzing "stylized" "refundable tax credits").

[20] See IRC § 38 (allowing, among others, "the research credit determined under section 41(a)" "against the tax imposed by this section"), § 41 (defining a "credit for increasing research activities").

[21] See Sarah B. Lawsky, How Tax Models Work, 53 B.C. L. Rev. 1657, 1691 (2012) (requesting that the author be "explicit [about the model's] assumptions and limitations").

First, we consider start-up founders to be rational, risk-neutral[22] actors interested in maximizing the value of their firm.[23] As we apply it to start-up firm decisions, the expected value framework means that the firm makes the decision that maximizes the expected value of the firm. We do not consider agency costs.

Second, the path to success for a start-up firm requires the navigation of a series of milestones or break points.[24] We use two break points in our analysis. The first break point is the "phase one" receipt of external private financing, such as VC financing. The second break point is the "phase two" successful exit from private ownership, for example, through a strategic acquisition or the completion of an initial public offering.[25] When we describe "probability of success," we generally mean the joint probability of success at each of these break points.

[22] See Sarah B. Lawsky, On the Edge: Declining Marginal Utility and Tax Policy, 95 Minn. L. Rev. 904, 914–17 (2011) (explaining relationship between the risk neutrality and rational expected value model).

[23] This may not be a complete description of the motivations of start-up founders. Behavioral economics and norms surely have relevance. One theory, for example, is that Silicon Valley founders and venture capitalists fail to tax plan because "by temperament, if nothing else, the venture capitalist finds it easier to focus instead on the huge upside potential of the investment he touts." Bankman, note 13, at 1765. Nevertheless, we consider it reasonable to assume that a start-up founder is very interested in making money through an increase in the value of his or her firm. Some behavioral economics and social norm concerns—including path dependence and conformity to market norms—might prompt start-ups to overestimate planning costs or undervalue future income tax savings. These considerations would not weaken our argument that even if a rational start-up fully valued future tax savings, it would likely still not invest in tax planning to achieve such savings. Perhaps the most challenging behavioral economics consideration for purposes of our argument is, paradoxically, optimism bias, which is the very "temperament[al]" feature said to explain why start-up founders and investors ignore taxes. See id. Optimism bias could challenge our analysis because an unreasonably high estimate of likely future success might also lead to an unreasonably high estimate of the value of future income tax savings.

[24] The idea of successive break points is consistent with related literature. See, e.g., Rafael Repullo & Javier Suarez, Venture Capital Finance: A Security Design Approach, 8 Rev. Fin. 75, 79 (2004) (outlining three entrepreneurial stages: The "start-up stage," the "expansion stage," and the "cash-out stage").

[25] These two specific break points do not exhaust the descriptive possibilities, but rather are meant to capture the idea that a start-up must negotiate a series of obstacles (each under capital constraint) en route to profitability and success. In reality there are far more than two break points. For example, external investment comes in successive waves whose closing and terms depend on the achievement of certain prior goals. See Robert H. Keeley, Sanjeev Punjabi & Lassaad Turki, Valuation of Early-Stage Ventures: Option Valuation Models vs. Traditional Approaches, 5 J. Entrepreneurial and Small Bus. Fin. 115, 121-25 (1996) (proposing the valuation of early ventures using a multistage call option approach).

Third, start-ups are resource constrained.[26] They finance operations from a series of limited pools of capital, such as self-financing,[27] fee deferrals provided by law firms and other service providers,[28] and incremental stages of equity financing provided by VC and other investors. We only consider financial resources in this Article, but the insight applies more generally to resources such as the time and energy of the start-up firm's management. We have in mind firms that will spend their endowment in an effort to fund future profits.[29]

Fourth, the firm faces the task of completing each of its break point goals with a finite amount of time that depends on the firm's endowment and cash burn rate. This idea proceeds from the combination of a resource-constrained start-up that strives to achieve successive milestones. The firm's finite endowment translates, through a cash burn rate, to the firm's survival time, meaning the amount of time the taxpayer has to achieve the next stage of success. For example, the more time a start-up has to succeed at phase one, by securing VC or other external funding, the more likely the start-up will be successful. We consider probability of success to depend entirely on survival time, and assume that the cumulative probability of success increases with survival time.[30]

[26] In other words, we assume that they cannot readily access liquid credit markets or operating profits as reliable sources of capital. See, e.g., Hemel & Ouellette, note 8, at 334-38 (describing start-ups' capital constraint, analyzing information problems and other reasons for it, and defending the particular importance of ex ante benefits to support innovation in the case of capital-constrained start-ups). This idea of capital rationing presents a tension with the corporate finance rule of thumb that states that firms have sufficient access to capital markets to undertake all projects that return a positive net present value. See, e.g., Richard A. Brealey & Stewart C. Myers, Principles of Corporate Finance 113, 115-17 (4th ed. 1991). Some empirical research, however, supports the general proposition that "small and young firms" are more capital-constrained due to "asymmetric information in capital markets." Carlos Carriera & Filipe Silva, No Deep Pockets: Some Stylized Empirical Results on Firms' Financial Constraints, 24 J. Econ. Surveys 731, 735 (2010).

[27] See, e.g., Startups in Australia: From Lucky to Plucky, The Economist, Jan. 9, 2016, at 57-58 (reporting that Australian firm Atlassian was founded in 2002 with a credit card loan then equal to $5400 in U.S. dollars).

[28] See Susan C. Morse, Startup Ltd.: Tax Planning and Initial Incorporation Location, 14 Fla. Tax Rev. 319, 346 (2013) (reporting customary fee deferrals of $15,000 to $25,000).

[29] We would have to modify our model to account for firms with significant early operating profits.

[30] We do not account for variation in the likelihood of firm success based on the quality of a firm's management or business plan, or the strength of the economy at the time of its founding, or any other unstated potential factors in a firm's success.

Fifth, start-ups are risky.[31] It is understood that a start-up probably will fail. Popular culture tells the story of never profitable start-ups,[32] start-ups with contested claims to essential intellectual property,[33] and start-ups whose chance at VC financing perpetually hangs on a thread.[34] Start-up failure is an important part of the narrative.[35] Start-up failure is also an important part of the available empirical data. The joint probability of receiving VC financing, successfully completing an initial public offering, and becoming profitable is likely no more than 1%.[36]

Sixth, when an external investor, such as a VC firm, decides to back a start-up, the decision rests on the start-up's business plan. The start-up's tax plan does not affect the funding decision, not even on the margin. So too for the decision about whether a start-up can successfully complete an IPO. This view draws support from some interview evidence,[37] from the custom of valuing a start-up based on revenues, or even future revenues, and from the norm of valuing more mature

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31 Natalie Robehmed, What Is a Startup?, Forbes (Dec. 16, 2013, 8:42 AM), http://www.forbes.com/sites/natalierobehmed/2013/12/16/what-is-a-startup/ (describing widespread agreement that start-ups are inherently risky). "?A startup is a company working to solve a problem where the solution is not obvious and success is not guaranteed.'" Id. (quoting Neil Blumenthal, cofounder and co-CEO of Warby Parker). "?Startup is a state of mind . . . It's when people join your company and are still making the explicit decision to forgo stability in exchange for the promise of tremendous growth and the excitement of making immediate impact,'" Id. (quoting Adora Cheung, cofounder and CEO of Homejoy). See also Michael Lewis, The New New Thing: A Silicon Valley Story 86 (1999) (characterizing start-ups as "huge gambles on the future"). Alex Wilhelm argues that when a company gets too big—measured in revenue, employees, or value, not profit—it no longer deserves the "start-up" label. See Alex Wilhelm, What the Hell Is a Startup Anyway?, TechCrunch (Dec. 30, 2014), http://techcrunch.com/2014/12/30/what-the-hell-is-a-startup-anyway/.

32 See e.g., Lewis, note 31 (chronicling Netscape).

33 See The Social Network (Sony Pictures 2010) (chronicling Facebook).

34 See Silicon Valley (HBO Television Broadcast 2014-present) (chronicling fictional company Pied Piper).

35 For example, start-up failure themes inspire both conferences, see, e.g., About FailCon, Failcon, http://thefailcon.com/about.html (last visited Apr. 13, 2016) ("FailCon is a one-day conference for technology entrepreneurs, investors, developers, and the designers to study their own and others' failures and prepare for success."), and cartoonists, see, e.g., Randy Glasbergen (1998), www.glasbergen.com/?count=3&S=fail&submit=search (last visited Mar.22, 2016) ("Don't tell them we failed. Tell them we decided to temporarily postpone our success."); Sidney Harris, www.cartoonstock.com/cartoonview.asp?catref=shrn2640 (last visited Mar. 22, 2016) ("Startups Anonymous. 7 companies in 3 years."); John Bernard Handelsman, The New Yorker, Dec. 25, 2000, at 102 ("The Internet startup only had enough cash for one more day. But, miraculously, the money lasted for eight days, until more venture capital could be raised.").

36 See notes 51-54 and accompanying text.

37 See Morse, note 28, at 346-47 (reporting some venture capitalists' weak preference for Delaware incorporation).

private companies based on EBITDA, an operating income measure that does not consider taxes.[38]

B. Variables

In our analysis, we use the variables set forth below in Table 1.

Table 1
Variables Used in Model

CF	pretax cash flow
r	discount rate[39]
n	number of time periods in firm's existence
p	cumulative probability of firm success
	p_{base} refers to base case business investment[40]
	$p_{tax\ plan}$ refers to tax planning case[41]
	p_{inv} refers to independent probability of success in phase one, securing external financing
	$p_{exit/inv}$ refers to probability of success in phase two, successfully exiting from private ownership, conditional on securing external financing in phase one
t	expected income tax rate
	t_{base} refers to tax rate in base case business investment
	$t_{tax\ plan}$ refers to tax rate in tax planning case
E	resource endowment of a firm
I	cost of tax planning investment
B	endowment cash burn rate of a firm, per unit of time
T	a firm's survival time, which is equal to $\frac{E-I}{B}$
λ	lambda, a rate parameter describing a firm's probability of success over time

C. Tradeoff Between Probability of Success and Tax Planning

Consider a start-up firm that faces the choice of whether or not to invest in tax planning. If it makes the investment, it will gain the advantage of a lower income tax rate on any future profits. But it will

[38] This premise that business performance is exogenous to tax planning is also consistent with the idea of modifying the value of a firm or its cost of capital with an adjustment for an interest deduction or other tax shield. See, e.g., Brealey & Myers, note 26, at 424-33 (discussing different approaches to valuing interest deductions and other tax shields as adjustments to the value of a firm).

[39] We are able to avoid questions about what discount rate to use because our manipulation cancels out this variable. See Pablo Fernandez, Valuing Companies by Cash Flow Discounting: Ten Methods and Nine Theories, 33 Managerial Fin. 853, 853 (2007) (highlighting examples of commonly used discounting methods).

[40] p_{base} denotes the overall probability of success in the base case, and therefore equals the joint probability of phase one and phase two success. In Part II, we use p_{base} without separating its phase one and phase two components. In Part III, we disaggregate p_{base} and separately state its components.

[41] $p_{tax\ plan}$ denotes the overall probability of success in the tax planning case, and therefore equals the joint probability of phase one and phase two success. In Part II, we use $p_{tax\ plan}$ without separating its phase one and phase two components. In Part III, where we illustrate our model with hypothetical start-up facts, we disaggregate $p_{tax\ plan}$ and separately state its components.

also face the disadvantage of a lower probability of success, because it will have spent capital on tax planning rather than business investment.

We assume that the start-up makes operational decisions that maximize the expected present value of its after-tax cash flows. This expected present value is a function of the firm's pretax cash flows, *CF*, discount rate, *r*, number of periods it is in operation, *n*, tax rate, *t*,[42] and probability of generating the cash flows, *p*. In other words, the expected value of a firm equals the present value of its future after-tax cash flows, or

$$\left(\frac{CF_0}{(1+r)^0} + \cdots + \frac{CF_n}{(1+r)^n}\right)(1-t)\,(p).$$

If the start-up does not tax plan, we refer to this as the base case. The expected value of the firm in the base case equals

$$\left(\frac{CF_0}{(1+r)^0} + \cdots + \frac{CF_n}{(1+r)^n}\right)(1-t_{\,base})\,(p_{\,base}).$$

If the start-up chooses to invest in tax planning, we refer to this as the tax planning case. The expected value of the firm in the tax planning case equals

$$\left(\frac{CF_0}{(1+r)^0} + \cdots + \frac{CF_n}{(1+r)^n}\right)\left(1-t_{\,tax\,plan}\right)\left(p_{\,tax\,plan}\right).$$

Tax planning will cause $t_{tax\ plan} < t_{base}$. Also, because of our resource constraint assumption and the assumption that the firm will be valued based on pretax cash flows, we assume that the probability of success in the tax planning case will be less than the probability of success in the base case, or in other words that $p_{tax\ plan} < p_{base}$.

We propose that the firm will choose to make an investment in tax planning only if the value of the firm after tax planning exceeds the value in the base case without tax planning. Put differently, the firm will accept a reduction to the expected probability of success in the tax planning case only if it is more than offset by the expected future reduction in income taxes. This analysis supports our first proposition, below:

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42 We do not adjust for investor-level taxes under the assumption that the after-tax (1 – *t*) ratio applicable as a result of investor-level taxes would be the same in either scenario. We assume that investors in start-ups are equity investors, which is consistent with theoretical and empirical work. See, e.g., Casamatta, note 6, at 2061 (noting that theoretical prediction that venture capital investor will own common or preferred equity or convertible debt squares with empirical observations). Interest deductions provide a tax benefit to profitable firms. Cf. Franco Modigliani & Merton H. Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, 48 Am. Econ. Rev. 261, 272-73 (1958). But interest deductions are of less interest to a start-up firm in a net operating loss position. Cf. Allen, note 10, at 1 (finding that 78% of VC-backed IPO firms have an NOL at the time of issuance).

Proposition 1: A start-up has an incentive to invest in tax planning if the following relationship holds:

$$\frac{1 - t_{tax\,plan}}{1 - t_{base}} > \frac{p_{base}}{p_{tax\,plan}}.$$

We provide the proof for Proposition 1 in the Appendix.

D. Survival Time: A Function of Money

Above we argued that probability of success and the change to probability of success as a result of tax planning predict whether a rational capital-constrained firm will invest in tax planning. But what determines probability of success? Of course, many different factors are relevant, including management quality and luck, to name two. We leave these factors aside for the most part here, although we do consider the successful navigation of prior milestones later in this Article when we explore the different results of our analysis depending on whether or not a start-up firm has received VC financing.

Our purpose here is to explore the relationship between the probability of success, time, and money. We suggest here that the probability of success is a function of time, which in turn is a function of money. The more money a capital-constrained firm has, the more survival time it has. In particular, we suggest that a firm's endowment translates linearly to the firm's survival time, meaning the finite amount of time that the firm has to reach the next milestone—whether phase one securing of external financing or phase two successful exit from private ownership. We translate a firm's endowment to its survival time by dividing the endowment, E, by the firm's cash burn rate per time period, B. If the firm invests I in tax planning, its survival time equals the difference between the endowment, E, minus the tax planning investment, I, divided by the cash burn rate, B. In other words, $T = \frac{E-I}{B}$

If a firm does not invest in tax planning, as in the business investment base case, then $I = 0$ and the firm's survival time equals $\frac{E}{B}$. Thus, each of E, I, and B affects T, the firm's survival time.[43]

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[43] Partial derivatives show the relationship between survival time T and each of the input variables, E, I, and B:

$\frac{dT}{dE} = \frac{1}{B}$, so survival time increases with an increase in endowment at a rate of $\frac{1}{B}$.

$\frac{dT}{dI} = -\frac{1}{B}$, so survival time decreases with an increase in the cost of tax planning at a rate of $-\frac{1}{B}$.

$\frac{dT}{dB} = -\frac{E-I}{B^2}$, so survival time decreases with an increase in cash burn rate at a rate of $\frac{E-I}{B^2}$.

E. Distribution of the Probability of Success over Time

The relationship between firm survival time and probability of success depends not only on the total survival time, but also on the shape of the probability function. In other words, we are interested in how the firm's cumulative probability of success increases over time. One might imagine a number of possible shapes for this probability function. For instance, it could be uniform, indicating an equal probability of success in each subpart of the firm's survival time. Or, the curve could be log normal, indicating a sharply increasing discrete probability of success early in the firm's life and then a bunching of the highest discrete probabilities of success at some set of time periods starting relatively soon after the firm's founding. Other functional forms are also possible.[44]

We elect here to use a negative exponential form for the shape of the function that describes the probability of firm success over time. We define cumulative probability of success at time T as $1 - e^{-\lambda T}$. The variable λ is a rate, or intensity, parameter. For our purposes, it describes the expected number of successes that will occur per unit of time. In the context of the negative exponential distribution, it approximates how quickly the discrete probability of success decreases over time.[45]

The negative exponential function approximates the probability distribution that we see in the data we collect about both start-up firm success in phase one, where a start-up seeks external financing, and in phase two, where a start-up seeks to exit from private ownership. Negative exponential functions are a familiar feature of science literature.[46] They describe, among other processes, the phenomenon of the decay of a radioactive material.[47] In statistics, the use of negative exponential functions has been used to model lifetimes.[48]

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44 Some alternative functional forms, including the log normal form, require the estimation of more than one parameter.

45 See, e.g., John C.B. Cooper, The Poisson and Exponential Distribution, 37 Mathematical Spectrum 123, 123-24 (defining λ); see generally George Casella & Roger L. Berger, Statistical Inference 98-102 (2d ed. 2002) (describing negative exponential and related forms of probability distributions).

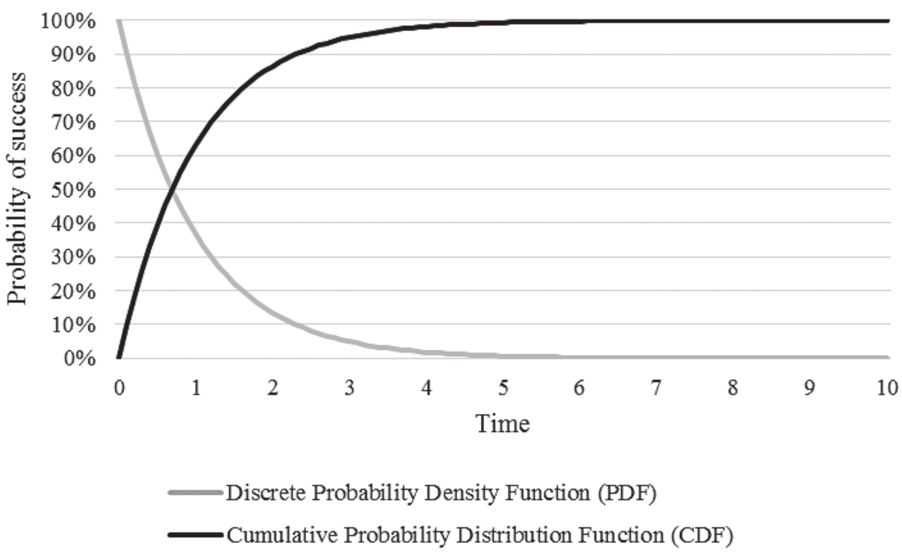
46 E.g., Albert W. Marshall & Ingram Olkin, A New Method for Adding a Parameter to a Family of Distributions with Application to the Exponential and Weibull Families, 84 Biometrika 641, 641 (1997) ("Exponential distributions play a central role in analyses of lifetime or survival data, in part because of their convenient statistical theory, their important 'lack of memory' property and their constant hazard rates.").

47 E.g., Eric B. Norman, Stuart B. Gazes, Stephanie G. Crane & Dianne A. Bennett, Tests of the Exponential Decay Law at Short and Long Times, 60 Physical Rev. Letters 2246, 2246 (1988) (referring to the "exponential nature of . . . radioactive decay").

48 See, e.g., José Mata & Pedro Portugal, Life Duration of New Firms, 42 J. Indus. Econ. 227, 228 (1994) (studying the survival of new Portuguese manufacturing firms).

Figure 1 shows both the cumulative probability of success given an elapsed time (as the line marked "CDF") and the discrete probability of success for each time period (as the line marked "PDF"). The function for CDF is $p = 1 - e^{-\lambda T}$, as stated above. The function for PDF is $p = \lambda e^{-\lambda T}$. The discrete probability of success is highest in the earliest moments of a firm's efforts and declines exponentially throughout the period during which the firm pursues its goal. The cumulative probability of success increases over the survival time, although the incremental additions to the cumulative probability of success decrease with successive discrete time periods such as discrete months, consistent with the negative exponential form of the function.

FIGURE 1
Negative Exponential Distribution Function



Since we have expressed survival time, or T, as a function of endowment, E, investment in tax planning, I, and burn rate, B, we can also express the probabilities of success as a function of E, I and B. In the base case, the expected cumulative probability of success, or p_{base}, equals $1 - e^{-\lambda \frac{E}{B}}$. In the tax planning case, the expected cumulative probability of success, or $p_{tax\ plan}$, equals $1 - e^{-\lambda \frac{E-I}{B}}$.

When we use the above expressions to state the probability of success in the base case and in the tax planning case, we can restate Proposition 1 as follows:

Proposition 1 (restated): $\frac{1 - t_{tax\ plan}}{1 - t_{base}} > \frac{1 - e^{-\lambda \frac{E}{B}}}{1 - e^{-\lambda \frac{E-I}{B}}}$

We expect these ratios to each exceed one. That is, the tax rate in the tax planning case is less than the tax rate in the base case, which causes 1 - $t_{tax\ plan}$ to exceed 1 - t_{base}. Also, the probability of success in the base case exceeds the probability of success in the tax planning case. Here, the probabilities of success are explained in their negative exponential form. We can also say that the ratio between the two probabilities of success will exceed one because endowment, or *E,* is greater than endowment less the investment in tax planning, or *E* – *I*, which causes $e^{-\lambda\frac{E}{B}}$ to be less than $e^{-\lambda\frac{E-I}{B}}$ and $1 - e^{-\lambda\frac{E}{B}}$ to exceed $1 - e^{-\lambda\frac{E-I}{B}}$.

F. How Much Should a Start-up Spend on Tax Planning?

Under the negative exponential function, the cumulative probability of success increases over time, but at a decreasing rate. This means that the marginal effect of a reduction in survival time, or *T*, will be greater if the lost survival time comes earlier in a firm's life. One factor that can cause a reduction in survival time is *I*, the cost of an investment in tax planning. *I* is the factor that is the output of our later illustration in Part IV of this Article. That illustration in turn is founded on Proposition 2:

> *Proposition 2*: A start-up firm has an incentive to invest in tax planning if:
> $$I < E + \frac{B}{\lambda} * ln\left(1 - \left(\left(1 - e^{-\lambda\frac{E}{B}}\right) * \frac{(1 - t_{base})}{(1 - t_{tax\ plan})}\right)\right).$$

We provide the proof for Proposition 2 in the Appendix.

Proposition 2 generates a value of *I* that will be less than *E*. This is because the natural log term, $ln\left(1 - \left(\left(1 - e^{-\lambda\frac{E}{B}}\right) * \frac{(1 - t_{base})}{(1 - t_{tax\ plan})}\right)\right)$, will be the natural log of a number between zero and one and therefore will be negative.[49] The value of *I* depends on the variables *E*, *B*, λ, and on the ratio between the after-tax factors. Larger values of *E* translate to larger values of *I*, both because *E* itself is larger and because a larger *E* produces a

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[49] The probability of success, or $1 - e^{-\lambda\frac{E}{B}}$, is a number between zero and one. The ratio of the after-tax factors, or $\frac{(1 - t_{base})}{(1 - t_{tax\ plan})}$, is also a number between zero and one. As a result, the product of the probability of success and the ratio of the after-tax factors will be a number between zero and one; and one minus that product will also be a number between zero and one. This means that the natural log term will equal the natural log of a number between zero and one, and that the natural log term will therefore be negative.

larger cumulative probability of success, which is given by $1 - e^{-\lambda \frac{E}{B}}$. Larger values of B translate to smaller values of I, because B is a numerator factor that increases the absolute value of the negative natural log term and because a larger B produces a smaller cumulative probability of success. Larger values of λ generally translate to larger values of I, because λ is a denominator factor that decreases the absolute value of the negative natural log term. Finally, if, for example, one holds t_{base} constant, then a smaller $t_{tax\ plan}$ translates to a larger I.[50]

III. A Hypothetical Start-up: Assumptions Used to Illustrate Model

A. *Assumptions: Probability of Success, Tax Rates in Base and Tax Planning Cases, and Endowment and Burn Rate*

The model we develop above investigates the relationship between money, time, and probability of success in the base case and in the tax planning case. Our idea is that if a resource-constrained firm spends money on tax planning, it cannot spend money on otherwise advisable business investment. This translates to a loss of survival time, meaning a decrease in the time available to the firm to successfully navigate the next milestone. The loss of survival time in turn means a decrease in the probability of the firm's success. Although the income tax benefit obtained from tax planning may encourage the firm to enter into the tax planning investment, the reduced probability of success discourages the tax planning investment. The firm must balance the advantage and the disadvantage and only faces a net incentive to make the investment if the benefit outweighs the cost.

This Part presents assumptions about a start-up firm's money, time, probability of success, and tax rate differentials. We build these assumptions from available empirical data. Then, in Part IV, we apply the model shown in Part II using the empirical assumptions developed in this Part to suggest how much a start-up might be willing to invest in tax planning.

The outcome of our development of empirical assumptions and our illustration of our model is a prediction of how much a start-up firm would invest to obtain a significant reduction in the income tax rate eventually due on any profits. In other words, the variable we are interested in is I. To illustrate I, we assume different concrete figures

[50] A lower $t_{tax\ plan}$ means that the after-tax factor ratio is a smaller number. This in turn means that the natural log term will calculate the natural log of a larger number. The natural log of a larger number (though still the natural log of a number between zero and one) will be closer to zero. It will be a negative number whose absolute value is smaller. This less negative value for the natural log term supports a larger E.

for the other variables in our model. In particular, we make assumptions (grounded to the extent possible in empirics) for probability of success in a start-up firm's base case and tax planning case, for a firm's tax rates in the base case and tax planning case, for a firm's endowment, *E,* and cash burn rate, *B*.

When we discuss probability of success in this and the following Part, we separately consider the navigation of each of the break points that we assume a start-up must navigate. These break points are phase one receipt of external financing, and phase two exit from private ownership. We state a precise negative exponential probability distribution over time for each of phase one and phase two based on data that we collect.

As stated in Table 1, we use p_{inv} to indicate independent probability of success in phase one. For the phase one probability, we must adjust also for the probability of success in phase two, since the probability of interest in phase one is a joint probability: The tax benefits we are interested in will materialize only if the start-up also successfully navigates phase two. We use $p_{exit/inv}$ to denote the probability of success in phase two, conditional on achieving success in phase one. Thus the joint probability of success in phase one equals $p_{inv} * p_{exit/inv}$.

B. Probability of Success, Phase One

An often-cited figure for the overall probability of successfully obtaining VC financing, a typical form of external financing for start-ups,[51] is approximately 1%. Different sources arrive at the 1% figure in different ways. A Kauffman Foundation study shows a 0.73% chance of VC financing, for example, based on survey data in which respondent firms include new firms with modest ambitions, such as local professional services or retail businesses.[52] However, not all of these firms would realistically be possible candidates for VC financing. Other data only uses industries that a start-up firm with global ambitions is likely to belong to. One set of data shows that of all computer, electronics, and telecom firms in existence between 1981

51 Paul Alan Gompers & Josh Lerner, The Venture Capital Cycle 3 (2d ed. 2004) (explaining that external funding for a start-up typically means investment by a venture fund that makes relatively long-term investments in "industries with a great deal of uncertainty").

52 The Kauffman project surveyed randomly selected firms founded in 2004, and oversampled high-technology businesses. See Kauffman Found., An Overview of the Kauffman Firm Survey: Results from the 2004-2008 Data 2-3 (May 2010). Of the 3564 firms studied, 26, of 0.73%, received venture capital financing. See id. at 11 tbl.4.

and 2005, between 1% and 2% received VC financing.[53] Other reports of a 1% chance of VC financing arise from back-of-the-envelope methodology resting on information such as VCs' reports of the proportion of business plans they accept.[54]

We use a relatively optimistic estimate of the likelihood of VC financing. Specifically, we use the proportion of "high-technology" firms that responded to the Kauffman survey that receive VC financing. This proportion is 4.4%.[55] The high-technology firms included in this category are those that list Standard Industrial Classification, or SIC, codes relating to the businesses of chemicals and pharmaceuticals, industrial equipment, electrical equipment, and instruments.[56]

The shape of the phase one negative exponential function is based on data we collected about the first date of VC financing for 21,846 companies in the VentureXpert database.[57] We find that the probability of receiving VC financing by time T, conditional on receiving VC financing at some point, can be expressed as $1 - e^{-0.024T}$.[58] This result is also shown below in Figure 2.

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[53] Manju Puri & Rebecca Zarutskie, On the Life Cycle Dynamics of Venture-Capital and Non-Venture-Capital-Financed Firms, 67 J. Fin. 2247, 2257 (2012) (reporting "1.04% for Computers, 1.20% for Electronics, and 1.64% for Telecom.").

[54] See, e.g., Dileep Rao, Why 99.95% of Entrepreneurs Should Stop Wasting Time Seeking Venture Capital, Forbes (July 22, 2013, 10:00 AM), http://www.forbes.com/sites/dileeprao/2013/07/22/why-99-95-of-entrepreneurs-should-stop-wasting-time-seeking-venture-capital/#10ace8cf296d ("VCs finance only about one or two ventures out of 100 business plans they see."); Henry Blodget, Dear Entrepreneurs: Here's How Bad Your Odds of Success Are, Business Insider (May 28, 2013, 11:03 AM), http://www.businessinsider.com/startup-odds-of-success-2013-5 (making a rough prediction of a 0.4% success rate based on applications to and participants in a leading start-up incubator).

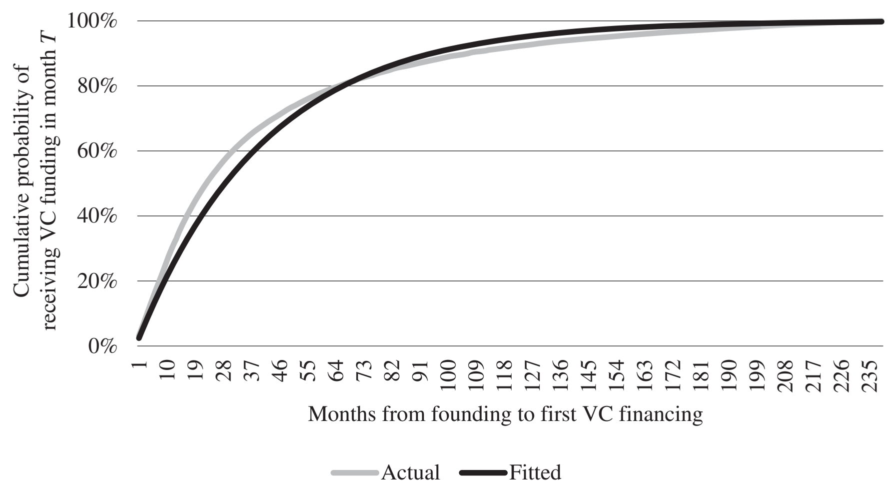
[55] We calculated the 4.4% figure based on the Kauffman data (covering survey years 2004-2008), available online at http://www1.kauffman.org/kfs/.

[56] Kauffman Found., note 52, at 19 (listing as high technology SIC codes 28 (chemicals and allied products), 35 (industrial machinery and equipment), 36 (electrical and electronic equipment), and 38 (instruments and related products)).

[57] Our data set begins with all U.S.-based companies that received VC financing as indicated in the SDC VentureXpert database. Thomson Reuter's VentureXpert, available through Thomson One, http://vx.thomsonib.com/NASApp/VxComponent/NewMain.htm (last visited Mar. 1, 2016). We eliminate firms with missing founding dates or date of first VC investment, or date fields with erroneous information. We further eliminate all firms where the founding date is after the date of first investment, the difference between founding date and date of first investment is greater than twenty years, and the founding date is in the same month as the first date of VC investment. This results in the identification of 21,843 firms that received VC investment between January 1, 1990 and August 31, 2015. The first funding date variable in VentureXpert reflects the first time the firm receives any external funding. External funding may come from an established venture capital firm or from another investor, such as an angel investor who typically provides start-up financing in smaller amounts than VCs.

[58] We used statistical software to fit a negative exponential function to the observed data. This results in a λ of 0.024.

FIGURE 2
Cumulative Probability Distribution of Phase One Financing



As Figure 2 shows, the probability of funding increases over time, but it increases less with each successive month. This means that the highest per-month probability of success occurs in the first months after start-up formation. By month ten, 25% of firms that eventually will receive funding have received funding. The fiftieth percentile mark is at twenty-three months and the seventy-fifth percentile mark is at fifty-three months.[59]

We adjust the expression $1 - e^{-0.024T}$ to obtain p_{inv}, the independent probability of success in phase one, where the cumulative probability is the observed chance of securing external funding, 4.4%, rather than the 100% cumulative probability given by the function in the absence of the 4.4% factor.[60] When we do this, we can instead state the expression as $p_{inv} = 4.4\% * (1 - e^{-0.024T})$.[61]

59 Our findings are roughly consistent with a study by Manju Puri and Rebecca Zarutskie, who examine a sample of about 10,000 VC-financed firms and note the distribution of ages at the time of the first VC funding. They show that 96% of their sample receive financing within the first seven years of their life, and 74% within the first two years. Puri & Zarutskie, note 53, at 2290 tbl.A2 (listing firm age at time of VC financing). These results are also consistent with a probability distribution under which the probability of receiving funding is highest in the earliest portion of a firm's life cycle and decreases exponentially thereafter.

60 For large values of T, $e^{-\lambda T}$ approaches zero, and $(1 - e^{-\lambda T})$ approaches one.

61 Specifically, we use Bayes' Rule (which determines the probability of the occurrence of an event conditioned on some other event occurring) and multiply the observed density of firms that receive their first round of VC funding in month T, which represents the probability of getting funding in month T conditional on receiving VC investment, by the unconditional probability of a firm receiving VC financing, which we estimate at 4.4%.

Finally, we must consider the fact that the probability of success at phase one equals the joint probability of (1) successfully securing external investment and (2) successfully exiting from private ownership, conditional on having secured external investment. In other words, the joint phase one probability equals $p_{inv} * p_{exit/inv}$. As explained below, the largest possible value for $p_{exit/inv} = 25\%$. It is this 25% value for $p_{exit/inv}$ that we will use for our illustration in Part IV. So we define the joint phase one probability as $p_{inv} * p_{exit/inv}$, or $[4.4\% * (1 - e^{-0.024T})] * 25\%$, or $1.1\% * (1 - e^{-0.024T})$.

C. Probability of Success, Phase Two

The phase two milestone, as described above, is the successful exit from private ownership.[62] We rely for phase two probability on data on the probability that a venture-backed firm will return a profit to its founder, which is in turn derived from observations of payoffs when a firm exits private ownership through IPO or acquisition.[63] We use this data to proxy for the probability that a venture-backed firm will successfully exit from private ownership.[64] In particular, we rely on a meticulous study conducted by Robert Hall and Susan Woodward that considers almost every company that received venture financing in the United States between 1987 and 2008.[65] Hall and Woodward find that start-up founders receive a positive payoff only one-quarter of the time.[66] This forms the basis for our assumption that a venture capital-backed start-up has a 25% chance of a successful exit from private ownership.

With respect to the probability of success over time in phase two, we draw empirical data from the study of VC-backed firms by Puri and Zarutskie. They show the years from first VC financing to exit for three different outcomes: fail, acquired, IPO.[67] We assume that an

[62] It is possible that a firm could successfully complete an initial public offering, or IPO, and become profitable as a stand-alone firm. It is also possible that a firm could become the target in a strategic acquisition, and show profit either before or after the strategic acquisition. We do not have data that directly shows the profitability of start-up firms in both cases; indeed, it would be difficult to untangle the profitability of a start-up firm's business from the profitability of the larger firm in the strategic acquisition case. Cf. Puri & Zarutskie, note 53, at 2290 (noting that some acquisitions of start-ups may be start-up failures in disguise).

[63] See Robert E. Hall & Susan E. Woodward, The Burden of the Nondiversifiable Risk of Entrepreneurship, 100 Am. Econ. Rev. 1163, 1163 (2010).

[64] See id. at 1168 (explaining the calculation of exit values based on IPO and acquisition data).

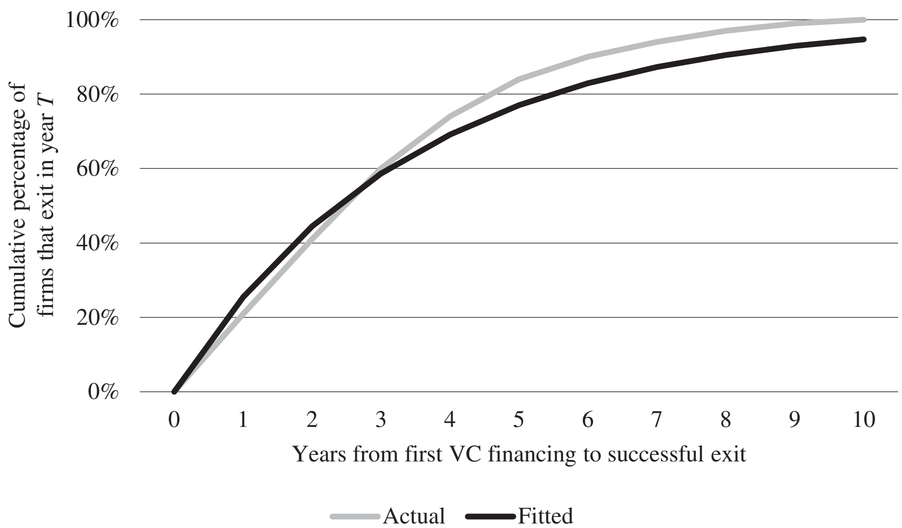
[65] Id.

[66] Id.

[67] Puri & Zarutskie, note 53, at 2270.

acquisition or IPO qualifies as a successful exit for our purposes.[68] The data can be fit to a negative exponential form, as shown. We find that the probability of successfully exiting by time *T*, conditional on successfully exiting for sure at some point and also conditional on having received venture capital financing, can be expressed as $1 - e^{-0.29T}$.[69] This result is also shown below in Figure 3.

FIGURE 3
Cumulative Probability of Phase Two Successful Exit



As Figure 3 shows, the cumulative probability of a successful exit increases with time, but it increases less in later years. As a corollary, the discrete probability of funding is higher in the years soon after VC financing, and declines over time after that, consistent with the negative exponential form of the function. The twenty-fifth percentile mark is between Years 1 and 2, the fiftieth percentile mark is at approximately Year 3, and the seventy-fifth percentile mark is at Year 4. The data can be fit to a negative exponential form, as shown.

We adjust the expression $1 - e^{-0.29T}$ so that the cumulative probability is the observed chance of a successful exit, 25%, rather than the 100% cumulative probability given by the function in the absence of the

68 Hall and Woodward treat acquisitions differently from Puri and Zarutskie, but we do not have reason to think that treating all acquisitions as successful exits changes the distribution of successful exits over time, which is the purpose of our analysis of the Puri and Zarutskie data.

69 We used statistical software to fit a negative exponential function to the observed data. This results in a λ of 0.029. The fit, however, is not as good as it is in the case of the probabilities over time for phase one success, in part because the data set only contains annual data.

25% factor.[70] After this adjustment, the modified expression is $p_{exit/inv} = 25\% * (1 - e^{-0.29T})$.[71]

D. Tax Rate Assumptions

We must also make assumptions about the tax rate faced by the firm in the base case and the tax planning case. We use an extreme case for our illustration presented in Part IV. In particular, we assume that a firm would face a 30% income tax rate on any future profits in the base case[72] and is able to reduce that rate to zero if it invests in tax planning. We envision a tax planning strategy that would reduce taxes for all firm profits, across the board, not just marginal earnings.[73] Assuming this rate reduction from 30% to zero allows us to consider the reduction in the probability of success that the firm would accept in exchange for the benefit of such a rate reduction. This is the relationship expressed in Proposition 1.

Of course, this is an extreme example, and perhaps it is not realistic to assume that a start-up could buy tax planning that reduces its tax rate to zero. But this tax rate reduction assumption allows us to establish an extreme case as a benchmark. The points we make also apply for less ambitious, more realistic tax planning goals.

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[70] For large values of T, $e^{-\lambda T}$ approaches zero, and $(1 - e^{-\lambda T})$ approaches one.

[71] Because a firm at phase two has by definition successfully navigated phase one, it is not necessary to calculate a joint probability for this final phase.

[72] The 30% starting rate draws support from available data. Much information available about corporate tax rates actually borne by firms is based on financial accounting reports. A GAO survey of available studies reports a range of 22% to 31.3% for effective tax rates drawn from company financials and based on various numerators (such as total book tax, actual tax reported, and cash book tax) divided by the denominator of pretax book income. These figures vary based on several factors, including whether loss firms are included in the sample and the years covered by the sample. U.S. Gov't Accountability Off., GAO-13-520, Corporate Income Tax: Effective Tax Rates Can Differ Significantly from the Statutory Rate 28 (2013), http://www.gao.gov/assets/660/654957.pdf. A recent contemporaneous and comprehensive study of financial accounting results reported by over 50,000 firms for a sample period from 1988 through 2012 reports a mean rate for 2012 of about 27% for cash taxes paid as reported for book purposes divided by pretax book income. See Scott D. Dyreng, Michelle Hanlon, Edward L. Maydew & Jacob R. Thornock, Changes in Corporate Effective Tax Rates over the Past Twenty-Five Years (Nov. 24, 2015), http://ssrn.com/abstract=2521497. These figures are based on financial accounting figures and may be closer to an upper bound rather than a mean estimate for actual taxes paid. When the GAO linked pretax book income figures to corporate tax returns to find taxes actually reported divided by book income, it found that an effective tax rate of between 12.6% and 15.3% for profitable filers and between 16.4% and 28.4% percent for all filers. U.S. Gov't Accountability Off., supra, at 15-16.

[73] But cf. John R. Graham & Lillian F. Mills, Using Tax Return Data to Simulate Corporate Marginal Tax Rates, 46 J. Acct. & Econ. 366, 366-67 (2008) (constructing marginal tax rates from tax return data because of importance of marginal tax rates on incremental corporate tax planning decisions).

E. Endowment and Burn Rate Assumptions

In order to illustrate the application of our model, which we do in the next Part, we must also make assumptions about firms' endowments and burn rates. Our objective in the illustration in Part IV is to show the different amounts a firm might be willing to spend on a tax planning investment, denoted by *I*, given the firm's otherwise available survival time in the base case. Survival time in the base case depends on endowment, *E*, and burn rate, *B*.

We do not know of any data available on endowments for start-up founders. However, during interviews with start-up founders and advisors, one of the authors heard repeatedly that law firms' practice of deferring $15,000 or more of fees was very valuable to start-up firms.[74] This suggests that start-up firms are not well-capitalized. Yet it is also the case that some start-up founders are serial founders who may have successfully sold their last venture and therefore are able to bring capital to the new one. To illustrate our model in phase one, we think it is useful to show how much start-ups with different endowments, under our assumptions, would pay for a particular tax rate differential. For phase one, we use endowments of $100,000,[75] $500,000, and $1 million[76] to illustrate our model, each with burn rates that translate to survival times of ten and twenty months.

To estimate reasonable values for endowments in phase two, we look to VentureXpert to collect data about the amount invested for each of the 22,000 or so firms reported in the data set that we used to fit the distribution of VC financing time in Figure 2. We found data about the investment amount for 19,345 of these firms, with a mean investment of about $3.5 million and a median investment of $13.1 million.[77]

The VentureXpert data reveal a 25th percentile investment amount of $4 million, a 50th percentile investment amount of $13.1 million, and a 75th percentile investment amount of $37.4 million. We use figures close to these, but rounded for convenience, for our illustra-

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74 See Morse, note 28, at 346 (reporting customary fee deferrals of $15,000 to $25,000).

75 An endowment of $100,000 would square with a high value placed on a $15,000 fee deferral by a law firm. It is roughly equivalent to the net wealth of a 50th percentile U.S. household according to Federal Reserve survey data. Jesse Bricker, Lisa J. Dettling, Alice Henriques, Joanne W. Hsu, Kevin B. Moore, John Sabelhaus, Jeffrey Thompson & Richard A. Windle, Changes in U.S. Family Finances from 2010 to 2013: Evidence from the Survey of Consumer Finances, 100 Fed. Res. Bull. 1, 37 (2014), http://www.federalreserve.gov/pubs/bulletin/2014/pdf/scf14.pdf.

76 One million dollars is roughly equivalent to the net wealth of a 90th percentile U.S. household according to Federal Reserve survey data. Id.

77 The difference between the mean and the median is driven by some very high outliers, as the maximum investment reported is $5.9 billion. See VentureXpert, note 57.

tion below as follows: $5 million, $15 million, and $40 million. We use burn rates that translate to survival times of one and two years.

IV. Illustrating the Model

A. *Phase One Illustration*

How much would a start-up pay to reduce its eventual income tax rate from 30% to zero? Below we show the results that our model suggests when a start-up is in phase one, seeking external financing. We express the results in terms of the dollars a start-up might invest in tax planning. Since the idea of resource constraint is broader than the financial capital constraint that we have referred to, it would also be possible to express the maximum investment in tax planning in some other unit, such as manager time or energy.

Table 2
Calculation of Tax Planning Investment Start-up Firm Has Incentive to Make in Phase One

E	B	T_{base} (months)	p_{base}	$p_{tax\ plan}$	$T_{tax\ plan}$ (months)	I
$100,000	$ 10,000	10	0.24%	0.17%	6.73	$ 32,000[78]
500,000	50,000	10	0.24%	0.17%	6.73	163,000
1,000,000	100,000	10	0.24%	0.17%	6.73	326,000
$100,000	$ 5,000	20	0.42%	0.30%	12.92	$ 35,000
500,000	25,000	20	0.42%	0.30%	12.92	177,000
1,000,000	50,000	20	0.42%	0.30%	12.92	353,000

[78] I can be calculated from Proposition 2 in the following manner:
Assuming inputs of:
E = $100,000
B = $10,000 per month
T_{base} = 10 months
t_{base} = 30%
$t_{tax\ plan}$ = 0%
I is calculated as:

$$I < E + \frac{B}{\lambda} * \ln(1 - \left(1 - e^{-\lambda\frac{E}{B}}\right) * \frac{(1-t_{base})}{(1-t_{tax\ plan})})$$

$$I < \$100{,}000 + \frac{\$10{,}000}{0.024} * \ln(1 - \left(1 - e^{-0.024*\frac{100{,}000}{10{,}000}}\right) * \frac{(1-0.3)}{(1-0.0)})$$

$$I < \$100{,}000 + \$416{,}667 * \ln(1 - (1 - e^{-0.24}) * 0.70)$$

$$I < \$100{,}000 + \$416{,}667 * \ln(.85)$$

$$I < \$100{,}000 - \$67{,}403$$

$$I < \$32{,}598$$

We round $32,598 down to the nearest $1000 and report I = $32,000 in 78 Table 2.

The first three rows of Table 2 show the hypothetical results for a new, phase one start-up whose endowment is enough to fund ten months of expenses. The rows' lines differ in the amount of the endowment: $100,000, $500,000 and $1 million. In each case, an endowment that will last for ten months gives the start-up a 0.24% chance of successfully navigating phase one by securing external funding. Setting the ratio of after-tax factors equal to the ratio of probabilities of success gives the result that the start-up would accept a reduction in the probability of success from 0.24% to 0.17% in order to obtain a tax rate reduction from 30% to zero. If the start-up only had enough endowment to cover 6.73 months of business spending, it would have a 0.17% probability of success. Thus it has an incentive to spend up to 10 minus 6.73, or 3.27 months of funding. This translates to about $32,000 if the start-up has an endowment of $100,000, $163,000 if the start-up has an endowment of $500,000, and $326,000 if the start-up has an endowment of $1 million.

The final three rows of Table 2 show the hypothetical results for a start-up whose endowment is enough to fund twenty months of expenses. The start-up has the opportunity to buy the same tax planning that would reduce its income tax rate on any future profit from 30% to zero. But for any given endowment, it should be willing to spend more on the tax planning that will give that result. This is because of the negative exponential shape of the probability distribution function in phase one. If the start-up has twenty months of funding, rather than ten months, then diverting some of those funds to tax planning will require the sacrifice of time that falls later in the start-up's survival time. The start-up has a lower discrete probability of obtaining external funding later in its survival time. So, it is willing to give up more time, which translates to more money, in order to tax plan.[79]

Could a start-up buy the complete elimination of all future income tax bills with the investment that it is willing to commit to such a project? The question is even more hypothetical than the exercise to this point, since such a strategy, let alone information about its pricing, is not readily available. If the strategy is understood to mean the establishment of an offshore structure by a U.S.-based firm, such as a structure that took advantage of patent box provisions in another jurisdiction, then at least some interview evidence suggests that merely organizing a Bermuda firm might cost "perhaps $30,000," compared to a cost of "$2000-$3000" for organizing a Delaware

[79] If a start-up had an even longer survival time, then it would be willing to spend even more on tax planning, given a certain endowment, on the theory that the months of business spending lost to tax planning do not carry high discrete probabilities of success.

corporation.[80] A structure with the audacious goal of eliminating all income tax presumably would cost far more. Also, such costs only represent organization costs, not maintenance costs. In the case of a multijurisdictional structure, many such costs-–such as the necessity of maintaining many intercompany contracts and two sets of lawyers-–are irreducible.

B. Phase Two Illustration

How much would a start-up pay to reduce its eventual income tax rate from 30% to zero after it has secured external financing? Below we show the results our model suggests when a start-up is in phase two, for example after the start-up has received VC financing.

TABLE 3
Calculation of Tax Planning Investment Start-up Firm Has Incentive to Make in Phase Two

E	B	T_{base} (years)	p_{base}	$p_{tax\ plan}$	$T_{tax\ plan}$ (years)	I
$ 5,000,000	$ 5,000,000	1	6.29%	4.41%	0.67	$ 1,657,000[81]
15,000,000	15,000,000	1	6.29%	4.41%	0.67	4,973,000
40,000,000	40,000,000	1	6.29%	4.41%	0.67	13,262,000
$ 5,000,000	$ 2,500,000	2	11.00%	7.70%	1.43	$ 1,825,000
15,000,000	7,500,000	2	11.00%	7.70%	1.43	5,475,000
40,000,000	20,000,000	2	11.00%	7.70%	1.43	14,601,000

The first three rows of Table 3 show the hypothetical results for a VC-backed phase two start-up with enough funding to cover one year of expenses. The final three rows illustrate the results for a phase two start-up with enough funding to cover two years of expenses. Such firms have an incentive to spend far more on tax planning compared to the new, phase one start-ups illustrated in Table 2. Future work might consider the possibility, for example, that take-up of incentives such as the R&D credit increases dramatically after a start-up obtains external financing.

The patent box example from the phase one illustration suggested that many start-ups would not bother to organize as non-U.S. corporations to benefit from patent boxes, because of the cost of planning. At first blush, this conclusion from phase one appears to turn out differently in phase two. Table 3's illustration suggests that a phase two start-up clearly has an incentive to spend $30,000 on

[80] Morse, note 28, at 345. The estimate of a Bermuda company's organization costs, however, was not based on any material experience setting up non-U.S. parent companies for U.S.-based start-ups.

[81] Numbers are rounded down to the nearest $1000.

lawyer's fees to set up an offshore company and additional resources to maintain the structure, if the structure would eliminate the 30% income tax rate.

It turns out, however, that the patent box strategy may often also be unattractive to a phase two start-up firm. This is because if the start-up firm has already organized as a C corporation under U.S. law, applicable anti-abuse rules make it quite expensive for the firm to "invert" into a non-U.S. corporation organized in the patent box jurisdiction.[82] For example, the shareholders who owned equity in the firm prior to the inversion generally would be liable for capital gains tax in an amount equal to the value of their stock minus their basis, if any.[83] Typically, the VC investment itself would result in a high value for the stock while the basis of much of the stock, certainly the stock held by the founders, would be very low. Thus in this patent box example, a higher endowment may make the start-up more willing to spend money on tax planning, but the higher endowment also makes the tax planning more expensive by increasing the capital gain toll charge.

C. Takeaways

Our analysis provides a different way to think about a capital-constrained firm's investment in tax planning. We propose that a capital-constrained firm such as a start-up does not think about investing in tax planning as a project of identifying all tax planning investments with positive net present value. Rather, because such a firm must operate with a limited endowment, it trades off the likely benefits of a possible investment in tax planning against the disadvantage of having less to spend on business investment. In our model, this reduction in working capital reduces the firm's expected probability of success. With assumptions that are grounded in at least some empirical data, we have illustrated how much a start-up might spend on tax planning that reduces its income tax rate from 30% to zero.

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[82] See, e.g., Shaviro, note 14, at 409-10 (discussing U.S. anti-inversion rules); Alison Bennett, Treasury Targets Inversions, Earnings Stripping in New Rules, Daily Tax Rep. (Apr. 5, 2016), http://www.bna.com/treasury-targets-inversions-n57982069452/ (discussing T.D. 9761, which addresses the anti-inversion rules under § 7874 and implements two previous notices aimed at curbing inversions).

[83] See Treasury Dep't, Corporate Inversion Transactions: Tax Policy Implications 7-9 (May 2002) (describing the capital gain recognition requirements for outbound transactions under § 367). This capital gain recognition requirement, applicable at least to shareholders, differs from the later efforts to curb inversion transactions through the 2004 enactment of § 7874. American Jobs Creation Act of 2004, Pub. L. No. 108-357, § 801, 118 Stat. 1418, 1562-66 (codified as amended at IRC § 7874, and disincentivizing inversions by focusing on the entity, rather than the shareholder level like § 367).

The first of two results that comes out of this illustration is that a start-up's appetite for tax planning during phase one, before any external investment has been secured, is extremely low. If a strategy costs several hundred thousand dollars to implement, even a well-endowed phase one start-up will not see the investment as worthwhile under the assumptions we outline. The analysis suggests, for example, that it is not surprising that U.S.-based start-ups do not incorporate in tax haven jurisdictions at the moment of their founding.

The tradeoff revealed by our model is applicable in other situations. For example, in the case of the nonrefundable R&D credit offered by current law, we should expect take-up to be greater when the advantage of reduced future income taxes outweighs the disadvantage of reduced working capital. Thus take-up should be smaller when firm endowments are smaller relative to the cost of claiming the credit.[84] Even in the case of a refundable tax credit, there is a tradeoff between the advantage of the credit and the disadvantage of less working capital. This is because there is a time lag-–perhaps a year or so—between the claiming of the credit and the receipt of a tax refund,[85] and so a firm that claims the credit sacrifices the use of some working capital for about a year. In contrast, an incentive that offers a payroll tax break, rather than an income tax break, can increase the cash in a firm's bank account very quickly, likely in weeks, because of the frequent deposit requirements for payroll tax payments. The prompt cash back feature of payroll taxes means that the disadvantage of losing access to working capital for a period of time likely will not be acute.

The second of two results is that a start-up has an incentive to spend far less in phase one, when it is still seeking external investment, than in phase two, after it has secured external investment. This is so for two reasons. First, the assumptions we use for endowment amounts following external investment are greater. Second, the start-up has a higher base case probability of success after securing external investment.

To return to the concrete example of patent box planning, our analysis suggests that a phase one U.S.-based start-up will not incur the tax planning expense of setting up in a patent box jurisdiction such as the United Kingdom, even if substantial income tax savings would result. But our analysis also suggests that a phase two U.S.-based start-up

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84 Costs include not only trivial compliance costs, but also deadweight loss and additional planning such as R&D expenditure studies.

85 The point is similar to the observation in the earned income tax credit context that the timing of the delivery of benefits matters to the working poor, who are a different category of resource-constrained taxpayer. See Anne L. Alstott, The Earned Income Tax Credit and the Limitations of Tax-Based Welfare Reform, 108 Harv. L. Rev. 533, 579-84 (1995).

might have an incentive to take advantage of a patent box strategy. A so-called inversion strategy is likely too expensive, in part because it would incur substantial capital gains based on the value of the firm, which is set by reference to the very VC investment that makes a larger investment in tax planning possible. But if the start-up firm could delay its incorporation choice until the VC investment had occurred, more take-up of a patent box innovation incentive should be expected.[86] This idea of an early-stage start-up preserving the option of later tax planning is consistent with some other work that suggests that a firm might first organize as an LLC treated as a partnership or disregarded entity for federal income tax purposes, and later shift to a corporation.[87]

Finally, the core insight of this Article is not limited to the case of income tax planning in start-ups. A resource-constrained firm may face the disadvantage of a reduced probability of success whenever it invests in a project that will not attract investors. Whether the investment commands cash resources or noncash resources, such as the attention of time or company management, the tradeoff we identify likely helps to motivate decisions. Perhaps such a capital-constrained firm will fail to purchase insurance because of this tradeoff, or decide not to invest in intellectual property protection, if it has reason to believe that these investments will not improve its chances of reaching the next phase of success. The analysis in this Article suggests that if policies aim to influence young start-up firms, the policies will have better success if they are cheap for firms to implement or, at least, cheap for firms to keep open the option of later implementation.

V. Conclusion

Government policies that aim to encourage innovation may target start-up firms, as these firms are thought to be better able to produce innovative or entrepreneurial results. We present a model that develops an important reason to expect low take-up of such policies, in par-

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86 The venture fund industry has proven itself capable of legal innovation in other contexts. See, e.g., John F. Coyle & Joseph M. Green, Contractual Innovation in Venture Capital, 66 Hastings L.J. 133, 160-70 (2014) (describing less expensive capital structure developments such as seed financing that support less capital-intensive start-ups); Ronald J. Gilson, Engineering a Venture Capital Market: Lessons from the American Experience, 55 Stan. L. Rev. 1067, 1084-85 (2003) (analyzing the classic common-and-preferred stock structure for a VC-backed start-up).

87 Cf. Cullen & Gordon, note 16, at 1480 (discussing the option value of a lower corporate tax rate on profits that can be claimed by later incorporation after initial organization as a pass-through in order to claim tax losses at higher individual rates). Provisions including the anti-inversion rule of § 7874, which extend to domestic partnerships acquired by foreign corporations, must be considered, but an LLC still provides more flexibility.

ticular income tax incentives, by start-up firms. We reason that a capital-constrained start-up will only invest in income tax planning if the advantage—a possible future increase in the value of the firm attributable to higher after-tax income–-exceeds the disadvantage of the reduction in the probability of success that comes from diverting resources from business spending to tax planning.

To illustrate our model, we develop assumptions grounded in empirical data to the extent possible about a hypothetical, rational start-up firm. The model shows that in many cases such a firm would not have an incentive to invest a material amount in tax planning, even if the planning would completely eliminate an otherwise applicable 30% income tax rate. However, later in such a firm's life, for example when its endowment and chance of success is greater as a result of external financing, the firm has a greater incentive to spend on tax planning. This suggests that firms might pursue less expensive, early tax planning that keeps open the ability to tax plan later, even though the empirical data does not yet provide robust evidence that start-ups in fact use such strategies.

APPENDIX
Proof of Propositions

Proposition 1: A start-up has an incentive to invest in tax planning if the following relationship holds:

$$\frac{1 - t_{\,tax\,plan}}{1 - t_{\,base}} > \frac{p_{\,base}}{p_{\,tax\,plan}}.$$

Proof: A firm will invest in tax planning if and only if

$$\left(\frac{CF_0}{(1+r)^0} + \cdots + \frac{CF_n}{(1+r)^n}\right)\left(1 - t_{\,tax\,plan}\right)\left(p_{\,tax\,plan}\right) >$$

$$\left(\frac{CF_0}{(1+r)^0} + \cdots + \frac{CF_n}{(1+r)^n}\right)\left(1 - t_{\,base}\right)\left(p_{\,base}\right)$$

This can be rewritten as:

$$\left(1 - t_{\,tax\,plan}\right)\left(p_{\,tax\,plan}\right) > \left(1 - t_{\,base}\right)\left(p_{\,base}\right)$$

or

$$\frac{1-t_{\,tax\,plan}}{1-t_{\,base}} > \frac{p_{\,base}}{p_{\,tax\,plan}}.$$

Proposition 2: A start-up firm has an incentive to invest in tax planning if

$$I < E + \frac{B}{\lambda} * \ln\left(1 - \left(\left(1 - e^{-\lambda\frac{E}{B}}\right) * \frac{(1 - t_{base})}{(1 - t_{tax\,plan})}\right)\right).$$

Proof:

Start with Proposition 1:

$$\frac{1-t_{\,tax\,plan}}{1-t_{\,base}} > \frac{p_{\,base}}{p_{\,tax\,plan}}$$

$$p_{tax\,plan} > p_{base} * \frac{(1 - t_{base})}{(1 - t_{tax\,plan})}.$$

Substitute the functional form for $p_{\text{tax plan}}$:[88]

$$1 - e^{-\lambda T_{tax\,plan}} > p_{base} * \frac{(1-t_{base})}{(1-t_{tax\,plan})}$$

$$-\lambda T_{tax\,plan} < ln(1 - p_{base} * \frac{(1-t_{base})}{(1-t_{tax\,plan})})$$

$$T_{tax\,plan} > \frac{\ln\left(1 - p_{base} * \frac{(1-t_{base})}{(1-t_{tax\,plan})}\right)}{-\lambda}.$$

Substitute the functional form for $T_{tax\,plan}$:

88 It is not necessary to multiply the functional form by a factor such as 1.1% to obtain $p_{\text{tax plan}}$ or p_{base} in Proposition 2, since such a factor will adjust both probabilities and cancel out.

$$\frac{E-I}{B} > \frac{ln(1- p_{base} * \frac{(1-t_{base})}{(1-t_{tax\ plan})})}{-\lambda}$$

$$I < E + B * \frac{\ln\left(1- p_{base} * \frac{(1-t_{base})}{(1-t_{tax\ plan})}\right)}{\lambda}.$$

Substitute the functional form for p_{base}:

$$I < E + B * \frac{ln(1-\left(1-e^{-\lambda\frac{E}{B}}\right) * \frac{(1-t_{base})}{(1-t_{tax\ plan})})}{\lambda}$$

$$I < E + \frac{B}{\lambda} * \ln(1 - \left(1 - e^{-\lambda\frac{E}{B}}\right) * \frac{(1-t_{base})}{(1-t_{tax\ plan})})$$